

Spatial Thinking: Why It Belongs in the Preschool Classroom

Calla Pritulsky, Caroline Morano,
and Rosalie Odean
University of Delaware

Corinne Bower
Temple University

Kathy Hirsh-Pasek
Temple University and The Brookings Institution,
Washington, DC

Roberta Michnick Golinkoff
University of Delaware

Children’s early spatial thinking abilities are predictive of their later STEM (science, technology, engineering, and mathematics) achievement. While research has primarily focused on spatial skills in the home environment, spatial learning can also occur in schools and in informal learning settings in the real world. Despite calls for implementation—as in the Common Core standards—spatial skills instruction is absent from most early education classrooms. The current article identifies key practices that can be implemented by preschool educators to help foster spatial learning. Adopting Chatterjee’s (2008) Framework of Spatial Thought and Language, which offers a typology for different aspects of spatial thinking, we argue that spatial skills should be taught early, using examples from the classroom to illustrate how research can be brought to life. We suggest ways to apply simple practices that preschool educators can use to improve children’s spatial skills, as well as describe installations designed to foster spatial thinking that can be implemented in some form in preschools.

What is the significance of this article for the general public?

This article addresses the absence of spatial skills instruction and playful learning opportunities in preschools by presenting the latest research and providing evidence-based examples of how preschools can integrate spatial language and gestures into everyday activities, as well as provide spatial classroom activities. Since spatial thinking develops early and predicts later success in STEM domains, it is imperative to bring spatial learning experiences into preschool classrooms.

Keywords: spatial thinking, preschool, gesture, toys, spatial language

We use spatial thinking each and every day of our lives. Leaving your bed to go to the kitchen requires navigational skills, putting your shirt over your head when getting dressed requires spatial visualization skills, and even figuring out

how to insert your bank card in the ATM requires mental rotation skills. Spatial thinking encompasses the ability to mentally rotate objects (e.g., thinking about how shoes must be manipulated to fit in a shoebox), navigate

Editor’s Note. Megan Sumeracki served as the action editor for this article.—MBK

 Calla Pritulsky, Caroline Morano, and Rosalie Odean, School of Education, University of Delaware; Corinne Bower, Department of Psychology, Temple University; Kathy Hirsh-Pasek, Department of Psychology, Temple University, and The Brookings Institution, Wash-

ington, DC; Roberta Michnick Golinkoff, School of Education, University of Delaware.

Rosalie Odean is now at the Department of Human Sciences, Ohio State University. Corinne Bower is now at College of Education, University of Maryland.

Correspondence concerning this article should be addressed to Calla Pritulsky, School of Education, University of Delaware, Newark, DE 19716. E-mail: calla.pritulsky@outlook.com

spaces, and understand relative sizes, locations, and orientations of objects (e.g., find the fruit next to the bananas; [Newcombe & Shipley, 2015](#)). Spatial skills are necessary not only for everyday tasks but for STEM (science, technology, engineering, and mathematics) achievement and later careers (e.g., [Wai, Lubinski, & Benbow, 2009](#)).

Correlational data, for instance, suggest associations between students' spatial skills and STEM learning (e.g., [Gunderson, Ramirez, Beilock, & Levine, 2012](#); [Hodgkiss, Gilligan, Tolmie, Thomas, & Farran, 2018](#)). Moreover, experimental data suggest a causal role of spatial skills in STEM learning (e.g., [Bower et al., in press](#); [Cheung, Sung, & Lourenco, 2019](#); [Gilligan, Thomas, & Farran, 2019](#)). Providing children with domain-general experiences in spatial thinking may bolster the cognitive foundations needed to learn about more abstract, STEM-related concepts.

In the United States, research has found sex differences (e.g., [Voyer, Voyer, & Saint-Aubin, 2017](#)) and socioeconomic status (SES) differences (e.g., [Jirout & Newcombe, 2015](#)) in the spatial skills of children and adults. Although some studies claim to find few gender differences between boys and girls (e.g., [Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2017](#)), others report differences in spatial skills that emerge in infant and preschool years (e.g., [Kotsopoulos, Zambrzycka, & Makosz, 2017](#); [Pruden & Levine, 2017](#); [Quinn & Liben, 2008](#)). Still, further research finds gender differences in some tasks but not others: [Harris, Hirsh-Pasek, and Newcombe \(2013\)](#) found gender differences in a mental rotation task but not in a mental folding task. In fact, recent work has shown that boys are making greater gains in mental rotation skills during prekindergarten than their female classmates ([Abad, Odean, & Pruden, 2018](#)). Additionally, high-SES children as young as age 3 already outperform their low-SES peers on block building and other spatial tasks ([Verdine et al., 2014](#)).

Fortunately, spatial thinking is malleable and can be taught at a young age (e.g., [Bower et al., in press](#)), rendering preschool an ideal place to lessen SES and sex differences in spatial skills. While the elementary school years can be riddled with tests, memorization, and homework, preschool is a unique time during which children are expected to learn the essentials—including

spatial thinking—through play. Forty-percent of 3-year-olds and 68% of 4-year-olds attend preschool in the United States ([McFarland et al., 2019](#)). While not all preschools are affordable for families, some states (Vermont, Florida, and the District of Columbia) have adopted universal pre-K programs starting at age 4, which allow children to enroll in fully funded preschool programs ([Barnett & Gomez, 2016](#)). In other states, programs such as Head Start provide another option. Head Start serves many low-income families; in 2018, 1,050,000 children in the United States enrolled in the program (“Head Start Program Facts: Fiscal year 2018,” 2018).

Preschools can provide a valuable opportunity for teaching children spatial thinking. Spatial tasks, such as block building and mental rotation, present children with a plethora of spatial input that can be conceptualized and discussed in numerous ways. During preschool, children can strengthen their spatial skills in several ways through (a) experiences with spatial toys (e.g., [Jirout & Newcombe, 2015](#)), (b) the language addressed to them ([Pruden, Levine, & Huttenlocher, 2011](#)), (c) the use of gestures used by adults in spatial contexts ([Cartmill, Pruden, Levine, & Goldin-Meadow, 2010](#)), and (d) engaging in activities that require navigation ([Foreman, Warry, & Murray, 1990](#)). Furthermore, spatial thinking predicts preschool children's school readiness by providing the framework needed to learn later spatial and mathematical skills, such as number, shape, and spatial knowledge, all which are included in the Common Core Standards for Kindergarten ([Mix & Cheng, 2012](#); [National Governors Association Center for Best Practices, 2010](#); [Verdine et al., 2014](#)). Despite the findings that spatial thinking predicts later achievement in STEM fields, spatial instruction is largely absent from preschool curricula, and SES differences are evident as early as age 3 ([Verdine et al., 2014, 2017](#)). Additionally, while there is a large body of research on the spatial skills of preschool children, there remains a gap in the literature on the role that preschool teachers can play in fostering children's developing spatial skills.

To address the absence of spatial skills instruction from preschools, the current article will present the latest research and provide examples of how the research can be applied in preschools through spatial language, gestures,

and the selection of classroom activities. The article adopts Chatterjee's (2008) Framework of Spatial Thought and Language, which posits several types of spatial information that characterize objects. *Intrinsic characteristics* refer to an object's shape, arrangement of parts, orientation, size, and how it moves, whereas *extrinsic characteristics* refer to an object's location relative to other objects or to a frame of reference, such as an apple being above a book (Chatterjee, 2008; Newcombe & Shipley, 2015). Intrinsic and extrinsic spatial relations can both be demonstrated statically and dynamically. For example, identifying shapes in a book is a *static* intrinsic task, as the spatial relations do not change, while putting a puzzle together is a *dynamic* intrinsic spatial task, as the spatial relations of the puzzle pieces are changeable.

The current article will use Chatterjee's (2008) framework to situate our discussion of spatial instruction in preschools within the larger literature on spatial skills. The following sections will review literature on the roles of spatial language and gesture in promoting spatial thinking and how we can harness these findings to support spatial thinking in classrooms. In addition to recommendations for classroom instruction, we will also discuss opportunities to infuse spatial skill learning into the preschool environment through creating smaller replicas of "Playful Learning Landscapes" (Hassinger-Das, Bustamante, Hirsh-Pasek, & Golinkoff, 2018).

Spatial Language Aids Spatial Thinking

The language children produce and hear from others is strongly linked to their developing understanding of spatial relations (e.g., Gentner, Özyürek, Gürçanlı, & Goldin-Meadow, 2013; Pruden et al., 2011; Verdine, Bungler, Athanasopoulou, Golinkoff, & Hirsh-Pasek, 2017). Spatial language includes words and phrases that indicate the dimensions of objects (e.g., "big"), the shape of objects (e.g., "rectangle"), or objects' spatial properties (e.g., "bent"; Pruden et al., 2011). In this article, we operationalize spatial language as referring to the comprehension and use of terms describing spatial relations, such as "above" or "below", and spatial features, such as "wide" or "narrow".

Through spatial language, parents and teachers convey numerous kinds of spatial informa-

tion. Adults can use nouns to refer to *intrinsic* properties of objects, such as their shape, and can use prepositions to orient objects within an *external* reference frame. Adults can discuss the shape of a rectangular magazine (i.e., intrinsic), while pointing out that the magazine is *on* the table (i.e., extrinsic). Spatial language can be either *static*, describing the stable intrinsic and extrinsic characteristics of objects (i.e., a triangle has three sides), or *dynamic*, referring to changes in intrinsic and extrinsic characteristics of objects (i.e., two right angle triangles can be rotated and fit together to form a square; Chatterjee, 2008; Newcombe & Shipley, 2015). These properties also appear in combination; an adult can explain that a stop sign is an octagon, noting the shape's intrinsic and static properties, for example. Likewise, an adult might point out a car to the right of the grocery store but, as the car moves, update the language to reflect the car's new spatial location (e.g., the car is in front of the store), reflecting both external and dynamic properties. Due to the gap in research on teachers' spatial language, we harness the findings from studies using parents to infer best practices for classroom implementation.

Polinsky, Perez, Grehl, and McCrink (2017) found that when parents used a shape language script, which included shape terms and part-whole relations, with their 4-year-old children in a block wall museum exhibit, children used more spatial language (words such as "in", "here", and "big") than when parents used a goal-directed script, which discussed building structures, without using spatial terms, or no script. Only the shape language script condition led to significant improvement in children's performance on puzzles from pre- to posttest. Further analyses revealed that children's use of spatial language in this block exhibit was predictive of their improvement on the puzzle. Even during a short intervention, parents' spatial language led children to use more spatial language, resulting in improved spatial task performance. Furthermore, Balcomb, Newcombe, and Ferrara (2011) found that knowledge of prepositions (like "in" and "out"), as measured by the MacArthur Communicative Development Inventory, was related to 16- to 24-month-olds finding a puzzle hidden under a foam tile in the floor. These spatial terms matter—when children complete spatial tasks, they perform better if they hear spatial language

than nonspatial language (Casasola, Bhagwat, & Burke, 2009; Loewenstein & Gentner, 2005). For example, Loewenstein and Gentner (2005) found that preschoolers were more accurate in mapping an object's location in one box to a corresponding location in a second box when they heard spatial words, such as "on", "in", and "under", compared to when they did not.

While short-term benefits of parents' use of spatial language is certainly promising, research on long-term effects of using spatial language is also telling about its power. Much of the spatial language that children hear comes from adults who talk to them during spatial play. In fact, Ferrara, Hirsh-Pasek, Newcombe, Golinkoff, and Lam (2011) found that parental spatial language is more frequent during spatial play, in particular with blocks, than other types of play. Pruden et al. (2011) videotaped and transcribed the spatial language 52 parents and their children used while interacting naturally at nine time points between 14 and 46 months. The cumulative number of spatial terms parents used across these time points predicted the cumulative number of spatial terms children used during the same time points. In turn, children's spatial language predicted their spatial problem solving at 54 months (Pruden et al., 2011).

The implications of these interventions demonstrate the influence of spatial language on spatial performance. While some speculate that spatial language is effective due to its direct effect on spatial knowledge, others have found support for an attentional hypothesis: Spatial language, when directed toward a relevant task, increases children's selective attention to spatial features and relations, leading to spatial learning (Miller & Simmering, 2018). By using spatial language during problem solving (e.g., "turn that puzzle piece around"), adults may guide children's attention to spatial features, leading to greater learning. Regardless of the specific mechanism involved, the use of spatial language with children facilitates the acquisition of spatial knowledge and skill.

There is wide variability in parents' spatial language use (Pruden et al., 2011). Pruden and Levine (2017) found that between 34 and 46 months, boys used significantly more spatial language than girls, a difference mediated by parental spatial language use between 14 and 26 months. Hearing more spatial language early

may have a lasting impact on children's spatial thinking such that they know more spatial terms than their peers. Moreover, in a study with a large, representative sample of children between the ages of 4 and 7, boys had a higher reported frequency of spatial play than girls, even when controlling for spatial performance (Jirout & Newcombe, 2015). Thus, gender differences in the frequency of spatial play were not due to differences in spatial ability; rather, the genders may have differential access and/or encouragement to engage with spatial toys.

Preschool teachers should make sure not only to provide but also to encourage spatial play with blocks and puzzles for both boys and girls (Costales, Abad, Odean, & Pruden, 2014). Furthermore, parents' and teachers' use of spatial language should occur early and often, both in the home and in schools, with spatial toys.

Play with spatial manipulatives, or toys such as shape sorters, blocks, and puzzles, is a prime opportunity to help foster spatial language production in both parents and children (e.g., Verdine et al., 2018; Zosh et al., 2015). Not only do spatial toys create a rich context for spatial language, but spatial materials also serve as another source of spatial information themselves, which adults can capitalize on through guided play (e.g., Fisher, Hirsh-Pasek, Newcombe, & Golinkoff, 2013). When adults engage in *guided play*, they let children direct the interaction while responding to the children's interests. In this way, adults scaffold children's learning, providing instruction and subtly guiding the interaction toward a learning goal (Weisberg, Hirsh-Pasek, & Golinkoff, 2013; Weisberg, Hirsh-Pasek, Golinkoff, Kittredge, & Klahr, 2016; Zosh et al., 2018). Spatial toys differ in the sort of information they can yield. They can provide children with intrinsic information, such as the shape, orientation, or size of a manipulative, but whether the input is static or dynamic varies depending on the type of toy (Newcombe & Shipley, 2015). Some materials provide intrinsic-static input that supports shape learning, such as shape sorters and shape books (Newcombe & Shipley, 2015), while other toys, like puzzles and blocks, offer intrinsic-dynamic information that helps develop mental rotation and transformation (Jirout & Newcombe, 2015). Moreover, the quality of spatial play is also important for the development of children's spatial thinking. For example, providing

corrective feedback with spatial language to preschoolers during structured puzzle play (e.g., “the circle goes above the rectangle”) can boost their spatial performance (Bower et al., *in press*). Thus, scaffolding preschoolers during spatial play by using shape names and spatial relational terms (e.g., “above”, “next to”) may facilitate their spatial thinking.

One opportunity to use spatial language with preschoolers is when learning about shapes, a common preschool topic (National Governors Association Center for Best Practices, 2010). Verdine et al. (2018) investigated how different shape toy versions affected parents’ and children’s use of spatial language during a naturalistic play session. Children produced more spatial language tokens and shape names when playing with unusual, alternate-shape toys (e.g., isosceles triangle) than when playing with more typical, standard-shape toys (e.g., equilateral triangle). Children used less language when playing with shapes on a tablet application than when playing with physical toys. Parents used more spatial language with physical toys than when standard shapes were presented on a tablet. Zosh et al. (2015) similarly found that traditional shape sorters prompted more parental spatial language with children than electronic shape sorters. Although tablets can lead to a decrease in talk between children and adults (Verdine et al., 2018), professional development targeted at teachers may help promote more effective use of spatial learning apps with children. Children’s learning from digital devices may be enhanced when accompanied by in-person social interactions with an adult (Dore et al., 2018; Eisen & Lillard, 2020). Future research should address how teachers can be prepared to implement technology (e.g., electronic applications) to support children’s spatial skill learning from an early age. The effectiveness and role of educational technology in the development of spatial thinking is just beginning to be investigated. A recent study by (Bower et al., *in press*) suggests that 3-year-old children profit from the use of digital puzzles to the same degree as concrete materials in a spatial training intervention. Early mathematics education has also found benefits in the use of tablets (Schacter & Jo, 2017). Thus, more work is needed in early spatial education to examine the potential benefits of digital interfaces versus traditional, tangible materials.

Zosh et al.’s (2015) study and Verdine et al.’s (2018) study have significant implications for designing shape activities in preschool. First, using concrete toys rather than electronic toys and tablets may increase children’s and adults’ use of spatial language. Electronic toys and tablets often boast “enhancements” or additional features like lights and sound effects; these “add-ons” may make electronic toys seem more enticing than traditional toys. However, these additional features are often not related to the learning goal of the toy and may serve as distractions from learning. Zosh et al. (2015) found that parents spoke about the same number of utterances overall to their children when playing with traditional and electronic shape sorters. However, when playing with electronic toys, parents used fewer unique words overall and fewer spatial terms. While playing with electronic shape sorters, parents’ language was more toy related and often focused on “enhancements,” for example, “What does that button do?” Whereas, when playing with traditional shape sorters, parents used more shape-related language, for example, “Which one is the circle? Where does the circle go?” Second, incorporating atypical, unfamiliar shape versions in classrooms would likely promote more spatial language. These unfamiliar shape instances help children identify the defining features of shapes. For example, seeing both isosceles and equilateral triangles may help children learn that the defining feature of a triangle is the presence of three angles and not that it has three equal sides. When choosing toys to include in the classroom, preschools should prioritize toys with a wide variety of shape categories, as well as atypical shape variants. Unfortunately, these irregular versions of shapes are largely absent from children’s educational materials, and children’s early shape knowledge appears to mimic what is and is not available for them to see (Resnick, Verdine, Golinkoff, & Hirsh-Pasek, 2016; Verdine, Lucca, Golinkoff, Hirsh-Pasek, & Newcombe, 2016).

Additionally, teachers can incorporate guided play, which has been linked to increased parental spatial talk (Ferrara et al., 2011). Ferrara et al. (2011) found that both 4.5-year-olds and parents used more spatial language when they played with blocks in a guided play condition, in which they were provided with photographs depicting steps to build either a garage or a

helipad, than when playing in a free play condition. The photographs of the prescribed building steps may have served as a reminder to parents to direct the conversation toward topics, such as spatial configurations, that elicit more spatial language. Further, the dyad may have benefited from the shared goal (Ferrara et al., 2011). Including spatial instruction in preschool curricula may provide early educators with the necessary reminder to help increase children's early exposure to spatial language.

The Use of Gesture to Promote Spatial Thinking

Gestures—movements made by the hands or arms that enrich or emphasize communication—are so common that we often do not give them a second thought (Hostetter & Alibali, 2007). While they may seem trivial, gestures are powerful learning tools, especially in the context of spatial thinking. For example, Austin and Sweller (2018) found that 3- and 4-year-old children who used both speech and gestures conveyed more navigational information than children who used speech alone (Austin & Sweller, 2018). Kita and Özyürek (2003) suggest that gestures provide a unique “interface,” supplementing units of speech with spatial representations that may be more accessible. For example, when children want an object and try to describe the object's location, it may be difficult to use spatial language such as, “I want the thing that is above the box and beside the bear.” Instead, gesture can translate the spatial information into a communicable unit by allowing children to simply point to the object, indicating its location.

Gestures themselves can be either static (deictic) or dynamic (iconic), depending on the gesture type (Chatterjee, 2008; Newcombe & Shipley, 2015). A *deictic* gesture, or pointing gesture, can be either intrinsic-static, such as pointing to the leg of a stuffed bear, or extrinsic-static, such as pointing to a stuffed bear on an out-of-reach shelf. *Iconic* gestures, such as making a rotation gesture to express how a chair spins, enact a concept or word. These can be intrinsic-dynamic, as in the chair example, or extrinsic-dynamic, as when making a movement gesture to show where a chair will be moved in relation to other objects in the room. As seen in these four examples, gestures are

extremely versatile tools to represent each of the four types of spatial representations.

Researchers have assessed the relationship between gesture and cognitive abilities, such as spatial skills, quite early in development (Goldin-Meadow & Alibali, 2013; Iverson & Goldin-Meadow, 2005). Parents' use of spatial utterances (e.g., “wide”) and gestures (e.g., spreading the hands apart) with 14-month-old children predicts the number of spatial types (i.e., unique dimensional adjectives, spatial features, or shape terms) children can produce in their own speech at 42 months old (Cartmill et al., 2010). Young, Cartmill, Levine, and Goldin-Meadow (2014) found that 4- and 5-year-olds benefited from using both spatial language and gestures—preschoolers performed better on jigsaw puzzles when using both gestures and spatial language.

Gesture and speech combinations are especially helpful in the spatial domain, which often includes components that are difficult to explain only using speech, especially for children. For example, explaining how to pack a car for vacation is far simpler when gestures are involved: Instructing someone to “turn the blue suitcase on its side and then rotate it 45 degrees” is much more complicated than simply pointing to the blue suitcase and gesturing to show how it should be rotated. Indeed, Sauter, Uttal, Alman, Goldin-Meadow, and Levine (2012) showed that children conveyed more information about a spatial layout when using both gesture and spatial language.

Another area of spatial skills research that has received a great deal of attention is mental rotation, or the ability to mentally imagine how an object would look if turned (Ehrlich, Levine, & Goldin-Meadow, 2006; Levine, Goldin-Meadow, Carlson, & Hemani-Lopez, 2018; Stieff, Lira, & Scopelitis, 2016). Ehrlich et al. (2006) found that preschoolers' performance on mental transformation tasks improved when they made gestures, such as moving their hands to show how two shapes could be rotated to fit together. Levine et al. (2018) examined children's gestures during puzzle play and emphasized an important distinction between different types of gestures. Children who were trained to gesture to indicate the movement of pieces performed better on mental rotation and transformation tasks than children who were trained to point to the perceptual features of objects. Ges-

tures may be more effective in different learning contexts, depending on how closely they align with a learning goal (Alibali & Nathan, 2018). In the mental transformation examples, the goal is to move pieces—therefore, movement gestures are best suited to the task.

Parents and educators can use gestures early and often when speaking about spatial information, which is often abstract and difficult for children to grasp. Research points to three key practices, including (a) closely aligning gestures with learning goals, such as using movement gestures when speaking about moving objects together, rather than pointing gestures; (b) combining gestures with speech to either supplement or enhance explanations; and (c) encouraging children to use gestures when speaking.

Although gesture is often produced spontaneously with speech, it is not a skill generally taught to preservice teachers or parents (Goldin-Meadow, 2015). Some children appear to naturally use gestures while engaging in spatial thinking, while others do not. In light of research supporting the efficacy of gesture, children should be encouraged to use gestures. At the most basic level, teachers and parents can encourage children when problem solving, with prompts such as, “Try using your hands,” or “Can you show me with your hands?” By simply reminding children that they can use their hands as tools to represent and understand concepts, they may be more likely to engage in gesture. Moreover, teachers should also produce effective gestures during instructional activities as it can help students’ learning (e.g., Alibali et al., 2013). One study (Bower et al., *in press*) found that providing corrective feedback with gesture to preschoolers during structured puzzle play (e.g., rotating their hand in a certain direction or tracing the outline of a shape to indicate the correct location and orientation of the puzzle piece) can boost low-income preschoolers’ spatial performance.

Environmental Installations to Promote Spatial Engagement

Opportunities for spatial learning can be infused in the actual architecture of the classroom. Playful Learning Landscapes is a movement that transforms urban spaces into

playful learning environments (Bustamante, Hassinger-Das, Hirsh-Pasek, & Golinkoff, 2018; Hassinger-Das et al., 2018; Playful Learning Landscapes Action Network, 2019). Parkopolis, a life-sized mathematical and spatial board game in a children’s museum, led to increased spatial and mathematical talk, as well as questions about spatial and mathematical topics, between adults and children (Bustamante et al., 2018; Morano, Bustamante, Schlesinger, Golinkoff, & Hirsh-Pasek, 2019). Another installation, Urban Thinkscape, transformed a bus stop in Philadelphia into a space for parent–child interaction and learning, with a puzzle wall and hidden figures in a metal design supporting spatial skills development (Hassinger-Das et al., 2018). Playful Learning Landscapes have multiple goals, including fostering interaction between adults and children, involving community members, and increasing children’s language and STEM skills (Bustamante et al., 2018). While originally designed for learning outside the classroom, these goals likewise complement preschool classroom learning.

When designing preschool environments, architects and school leaders can think about designs that incorporate playful learning in ways that will promote spatial thinking through interactions between teachers and children. For example, in Parkopolis, children casually talk about shapes with caregivers when standing on the shape mat, decorated with different-colored shapes. When designing preschool classrooms, educators might consider including shape designs painted on the floor—a simple, yet effective method for promoting spatial talk and shape learning. Including construction paper outlines of where toys go on the shelves not only can help children clean up but can also be a great opportunity to introduce more shapes into the classroom, as children can see the shapes that make up real objects. While not all preschools have the resources to engage in large-scale architectural transformation, teachers can use inexpensive methods, such as homemade signs with atypical shape exemplars (e.g., isosceles triangle), to promote spatial learning.

Promoting Spatial Navigation Through Play Spaces

Where Playful Learning Landscapes were used to increase talk and thinking about intrinsic spatial concepts, the classroom environment can be incorporated into curriculum to encourage thinking about larger-scale extrinsic spatial concepts. Spatial navigation is the ability to use landmarks, distance traveled, cognitive maps, and, for many adults, tools like maps or GPS to orient and move through space (Nazareth, Newcombe, Shipley, Velazquez, & Weisberg, 2019). Large-scale spatial activities during childhood, like sports, have been linked to later spatial abilities and can help mitigate sex differences in adult spatial activities (Nazareth, Herrera, & Pruden, 2013).

The preschool period is one that shows rapid improvement in children's spatial working memory and ability to use landmarks to find hidden objects in familiar spaces (Foreman et al., 1990). Liben, Moore, and Golbeck (1982) had preschoolers complete two furniture arrangement tasks, one using a scale model of their classroom with furniture models made of balsa wood and the other using the actual classroom and full-sized cardboard representations of the classroom furniture. Children showed a greater capacity to arrange the furniture models in their classroom than to use a scale model, representing the importance of full-scale spatial activities for young children.

Another important developing spatial skill is the ability to use scale models or maps to understand how to find something or navigate a space. Very young children have a poor understanding of scale; toddlers tend to try to use miniature toys or objects as though they are full-sized objects (DeLoache, Uttal, & Rosengren, 2004). However, experience with small-scale objects decreases the frequency of these errors (Rosengren, Schein, & Gutiérrez, 2010). Rosengren and colleagues (2010) introduced familiar types of objects in small-scale form (i.e., a miniature couch, slide, and toy car) into two classrooms. After 10 weeks experience with these toys, children made fewer scale errors, showing that experience with scaled objects can change even young children's understanding of the objects' affordances.

Preschool aged children are just beginning to understand how to use representative models of

their environments, navigate spaces on their own, and understand the relations between large-scale objects in familiar locations (e.g., DeLoache et al., 2004; Foreman et al., 1990; Liben et al., 1982). By creating spaces and activities where children can manipulate large objects, practice using maps and other scale models, and try finding things on their own, preschool teachers can support children's burgeoning extrinsic spatial skills.

Conclusions

Although spatial instruction is largely absent from formal early education curricula, playful instruction on spatial thinking can be easily embedded in preschools and possibly help address the spatial lags in children from low-income homes that already exist by age 3 (Verdine et al., 2014, 2017), as well as gender differences in spatial skills (Abad et al., 2018). Taking findings from research and using them in classrooms offer preschool educators multiple options for implementing spatial skills instruction. Simply using more spatial language with children has been linked to improved spatial skills (Pruden et al., 2011), and it matters not whether the effect is specific to spatial language or to the attentional effects spatial language invites (Miller & Simmering, 2018). Additionally, encouraging children of both genders to engage with more spatial toys such as blocks and puzzles is likely to improve children's spatial thinking. Increasing the use of gesture provides another avenue for educators, and although it is difficult to become conscious of using gestures, it can be prompted (Alibali et al., 2013). Teachers can try to use more spatial gestures when talking about spatial topics, such as gesturing to show children where they will walk for lunch. Educators can even encourage children to try to use their hands when engaging in spatial problem solving, as it has been shown that gesturing of this nature improves spatial performance (Ehrlich et al., 2006; Levine et al., 2018; Stieff et al., 2016). Lastly, the preschool classroom itself can serve as a learning tool. Teachers can use the classroom creatively by hiding objects and challenging children to use navigational strategies and spatial language to uncover objects (e.g., Foreman et al., 1990).

Playful Learning Landscapes is another way to increase spatial talk between adults and chil-

dren, and teachers can consider ways to enrich play areas with elements of these installations. Based on similar designs described by Haslinger-Das et al. (2018), installations such as painting a ruler on the floor with whole integers and fractions, adding atypical shape exemplars to posters, and offering large foam floor puzzles might improve children's spatial language use in the classroom. Incorporating elements from the Playful Learning Landscapes initiative can be inexpensive yet effective for children's spatial skill development.

One common theme seen in spatial language, gesture, and preschool classroom design is the importance of adult interaction with children. Children need adults to guide their learning—and while children learn best when they are given freedom to follow their own curiosities and interests, adult interaction is also essential for learning. While peer interactions are certainly influential during the preschool years, the importance of engaging, involved adults cannot be understated. Preschool is not the time for rote memorization and “drill-and-kill” but instead a time for play. Preschool teachers can use guided play to follow the child's lead and identify and utilize opportunities to infuse spatial learning in classroom activities (Weisberg et al., 2013; Weisberg et al., 2016; Zosh et al., 2018). For example, when a child is painting, a teacher may use spatial language by pointing out how a painted bird is above the ground. When a preschooler is playing house, a teacher may use gestures to demonstrate how to rotate a baby doll to fit into a crib. Even when a child is interacting with the classroom itself, such as playing on a mat filled with shapes of different sizes and dimensions, the teacher can name shapes and point out similarities and differences between different shape exemplars.

Teachers should keep three main practices in mind when bringing spatial learning into the classroom. First, spatial learning should be fun. The examples given in this article mention playful activities such as block building and puzzle play. These activities should be engaging for children, naturally encouraging the development of spatial thinking. Second, spatial learning should include an adult. Children may miss out on opportunities for spatial learning without the presence of adult scaffolding. Third, spatial learning should be integrated at every level of the classroom. Beyond occasional activities, the

actual classroom can be designed to facilitate spatial learning by adopting principles from initiatives such as Learning Landscapes.

Translating the research on spatial thinking into practice in the classroom is entirely feasible and can only support children's learning and enjoyment. Preschool educators can even consider novel ways to do so. Given that spatial thinking skills begin developing early—and predict later success in STEM domains from an early age (Verdine et al., 2014)—it is imperative to bring spatial learning experiences into preschool classrooms.

References

- Abad, C., Odean, R., & Pruden, S. M. (2018). Sex differences in gains among Hispanic pre-kindergartners' mental rotation skills. *Frontiers in Psychology, 9*, 2563. <http://dx.doi.org/10.3389/fpsyg.2018.02563>
- Alibali, M. W., Crooks, N. M., Young, A. G., Yeo, A., Ledesma, I., Nathan, M. J., . . . Knuth, E. J. (2013). Students learn more when their teacher has learned to gesture effectively. *Gesture, 13*, 210–233. <http://dx.doi.org/10.1075/gest.13.2.05ali>
- Alibali, M. W., & Nathan, M. J. (2018). Embodied cognition in learning and teaching. In F. Fischer, C. E. Hmelo-Silver, S. R. Goldman, & P. Reiman (Eds.), *International handbook of the learning sciences* (pp. 73–83). New York, NY: Routledge. <http://dx.doi.org/10.4324/9781315617572-8>
- Austin, E. E., & Sweller, N. (2018). Gesturing along the way: Adults' and preschoolers' communication of route direction information. *Journal of Nonverbal Behavior, 42*, 199–220. <http://dx.doi.org/10.1007/s10919-017-0271-2>
- Balcomb, F., Newcombe, N. S., & Ferrara, K. (2011). Finding where and saying where: Developmental relationships between place learning and language in the first year. *Journal of Cognition and Development, 12*, 315–331. <http://dx.doi.org/10.1080/15248372.2010.544692>
- Barnett, S., & Gomez, R. (2016, January). *Universal Pre-K: What does it mean and who provides it?* Retrieved from <http://nieer.org/2016/01/06/universal-pre-k-what-does-it-mean-and-who-provides-it>
- Bower, C., Zimmermann, L., Verdine, B., Toub, T. S., Islam, S., Foster, F., . . . Hirsh-Pasek, K. (in press). Piecing together the role of a spatial assembly intervention in preschoolers' spatial and mathematics learning: Influences of gesture, spatial language, and socioeconomic status. *Developmental Psychology*.

- Bustamante, A. S., Hassinger-Das, B., Hirsh-Pasek, K., & Golinkoff, R. M. (2018). Learning landscapes: Where the science of learning meets architectural design. *Child Development Perspectives*, *13*, 34–40. <http://dx.doi.org/10.1111/cdep.12309>
- Cartmill, E., Pruden, S. M., Levine, S. C., & Goldin-Meadow, S. (2010). The role of parent gesture in children's spatial language development. In K. Franich, K. M. Iserman, & L. L. Kei (Eds.), *Proceedings of the 34th Annual Boston University Conference on Language Development* (pp. 70–77). Somerville, MA: Cascadilla Press.
- Casasola, M., Bhagwat, J., & Burke, A. S. (2009). Learning to form a spatial category of tight-fit relations: How experience with a label can give a boost. *Developmental Psychology*, *45*, 711–723. <http://dx.doi.org/10.1037/a0015475>
- Chatterjee, A. (2008). The neural organization of spatial thought and language. *Seminars in Speech and Language*, *29*, 226–238. <http://dx.doi.org/10.1055/s-0028-1082886>
- Cheung, C.-N., Sung, J. Y., & Lourenco, S. F. (2019). Does training mental rotation transfer to gains in mathematical competence? Assessment of an at-home visuospatial intervention. *Psychological Research*. Advance online publication. <http://dx.doi.org/10.1007/s00426-019-01202-5>
- Costales, A., Abad, C., Odean, R., & Pruden, S. M. (2014). Spatial activities and manipulatives for early education classrooms. In W. G. Scarlett (Ed.), *The Sage encyclopedia of classroom management* (pp. 769–771). Thousand Oaks, CA: Sage.
- DeLoache, J. S., Uttal, D. H., & Rosengren, K. S. (2004). Scale errors offer evidence for a perception-action dissociation early in life. *Science*, *304*, 1027–1029. <http://dx.doi.org/10.1126/science.1093567>
- Dore, R. A., Hassinger-Das, B., Brezack, N., Valladares, T. L., Paller, A., Vu, L., . . . Hirsh-Pasek, K. (2018). The parent advantage in fostering children's ebook comprehension. *Early Childhood Research Quarterly*, *44*, 24–33. <http://dx.doi.org/10.1016/j.ecresq.2018.02.002>
- Ehrlich, S. B., Levine, S. C., & Goldin-Meadow, S. (2006). The importance of gesture in children's spatial reasoning. *Developmental Psychology*, *42*, 1259–1268. <http://dx.doi.org/10.1037/0012-1649.42.6.1259>
- Eisen, S., & Lillard, A. S. (2020). Learning from apps and objects: The human touch. *Mind, Brain, and Education*, *14*, 16–23. <http://dx.doi.org/10.1111/mbe.12224>
- Ferrara, K., Hirsh-Pasek, K., Newcombe, N. S., Golinkoff, R. M., & Lam, W. S. (2011). Block talk: Spatial language during block play. *Mind, Brain, and Education*, *5*, 143–151. <http://dx.doi.org/10.1111/j.1751-228X.2011.01122.x>
- Fisher, K. R., Hirsh-Pasek, K., Newcombe, N., & Golinkoff, R. M. (2013). Taking shape: Supporting preschoolers' acquisition of geometric knowledge through guided play. *Child Development*, *84*, 1872–1878. <http://dx.doi.org/10.1111/cdev.12091>
- Foreman, N., Warry, R., & Murray, P. (1990). Development of reference and working spatial memory in preschool children. *Journal of General Psychology*, *117*, 267–276. <http://dx.doi.org/10.1111/j.2044-8295.1998.tb02697.x>
- Gentner, D., Özyürek, A., Gürcanli, O., & Goldin-Meadow, S. (2013). Spatial language facilitates spatial cognition: Evidence from children who lack language input. *Cognition*, *127*, 318–330. <http://dx.doi.org/10.1016/j.cognition.2013.01.003>
- Gilligan, K. A., Thomas, M. S. C., & Farran, E. K. (2019). First demonstration of effective spatial training for near transfer to spatial performance and far transfer to a range of mathematics skills at 8 years. *Developmental Science*. Advance online publication. <http://dx.doi.org/10.1111/desc.12909>
- Goldin-Meadow, S. (2015). Gesture and cognitive development. In L. S. Liben, U. Müller, & R. M. Lerner (Eds.), *Handbook of child psychology and developmental science: Cognitive processes* (7th ed., pp. 339–380). Hoboken, NJ: Wiley. <http://dx.doi.org/10.1002/9781118963418.childpsy209>
- Goldin-Meadow, S., & Alibali, M. W. (2013). Gesture's role in learning and development. In P. D. Zelazo (Ed.), *The Oxford handbook of developmental psychology: Body and mind* (Vol. 1, pp. 953–973). New York, NY: Oxford University Press.
- Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. *Developmental Psychology*, *48*, 1229–1241. <http://dx.doi.org/10.1037/a0027433>
- Harris, J., Hirsh-Pasek, K., & Newcombe, N. S. (2013). Understanding spatial transformations: Similarities and differences between mental rotation and mental folding. *Cognitive Processing*, *14*, 105–115. <http://dx.doi.org/10.1007/s10339-013-0544-6>
- Hassinger-Das, B., Bustamante, A., Hirsh-Pasek, K., & Golinkoff, R. (2018). Learning Landscapes: Playing the way to learning and engagement in public spaces. *Journal of Research in Education Sciences*, *8*, 1–21. <http://dx.doi.org/10.3390/educsci8020074>
- Head Start program facts: Fiscal year 2018. (2018). *Head Start Early Childhood Learning and Knowledge Center*. Retrieved from <https://eclkc.ohs.acf.hhs.gov/about-us/article/head-start-program-facts-fiscal-year-2018>
- Hodgkiss, A., Gilligan, K. A., Tolmie, A. K., Thomas, M. S. C., & Farran, E. K. (2018). Spatial cognition and science achievement: The contribu-

- tion of intrinsic and extrinsic spatial skills from 7 to 11 years. *British Journal of Educational Psychology*, 88, 675–697. <http://dx.doi.org/10.1111/bjep.12211>
- Hostetter, A. B., & Alibali, M. W. (2007). Raise your hand if you're spatial: Relations between verbal and spatial skills and gesture production. *Gesture*, 7, 73–95. <http://dx.doi.org/10.1075/gest.7.1.05hos>
- Iverson, J. M., & Goldin-Meadow, S. (2005). Gesture paves the way for language development. *Psychological Science*, 16, 367–371. <http://dx.doi.org/10.1111/j.0956-7976.2005.01542.x>
- Jirout, J. J., & Newcombe, N. S. (2015). Building blocks for developing spatial skills: Evidence from a large, representative U.S. sample. *Psychological Science*, 26, 302–310. <http://dx.doi.org/10.1177/0956797614563338>
- Kita, S., & Özyürek, A. (2003). What does cross-linguistic variation in semantic co-ordination of speech and gesture reveal?: Evidence of an interface representation of spatial thinking and speaking. *Journal of Memory and Language*, 48, 16–32. [http://dx.doi.org/10.1016/S0749-596X\(02\)00505-3](http://dx.doi.org/10.1016/S0749-596X(02)00505-3)
- Kotsopoulos, D., Zambrzycka, J., & Makosz, S. (2017). Gender differences in toddlers' visual-spatial skills. *Mathematical Thinking and Learning*, 19, 167–180. <http://dx.doi.org/10.1080/10986065.2017.1328634>
- Levine, S. C., Goldin-Meadow, S., Carlson, M. T., & Hemani-Lopez, N. (2018). Mental transformation skill in young children: The role of concrete and abstract motor training. *Cognitive Science*, 42, 1207–1228. <http://dx.doi.org/10.1111/cogs.12603>
- Liben, L. S., Moore, M. L., & Golbeck, S. L. (1982). Preschoolers' knowledge of their classroom environment: Evidence from small-scale and life-size spatial tasks. *Child Development*, 53, 1275–1284. <http://dx.doi.org/10.2307/1129017>
- Loewenstein, J., & Gentner, D. (2005). Relational language and the development of relational mapping. *Cognitive Psychology*, 50, 315–353. <http://dx.doi.org/10.1016/j.cogpsych.2004.09.004>
- McFarland, J., Hussar, B., Zhang, J., Wang, X., Wang, K., Hein, S., . . . Barmer, A. (2019). *The condition of education 2019* (NCES 2019–144). Washington, DC: National Center for Education Statistics. Retrieved from <https://nces.ed.gov/pubsearch/pubinfo.asp?pubid=2019144>
- Miller, H. E., & Simmering, V. R. (2018). Children's attention to task-relevant information accounts for relations between language and spatial cognition. *Journal of Experimental Child Psychology*, 172, 107–129. <http://dx.doi.org/10.1016/j.jecp.2018.02.006>
- Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: Developmental and educational implications. *Advances in Child Development and Behavior*, 42, 197–243. <http://dx.doi.org/10.1016/B978-0-12-394388-0.00006-X>
- Morano, C., Bustamante, A., Schlesinger, M., Golinkoff, R. M., & Hirsh-Pasek, K. (2019, May). *What's Parkopolis? STEM questions in a children's museum*. Paper presented at the symposium for the Association for Psychological Science, Washington, DC.
- National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core state standards for mathematics*. Washington, DC: Author. Retrieved from <http://www.corestandards.org/Math/>
- Nazareth, A., Herrera, A., & Pruden, S. M. (2013). Explaining sex differences in mental rotation: Role of spatial activity experience. *Cognitive Processing*, 14, 201–204. <http://dx.doi.org/10.1007/s10339-013-0542-8>
- Nazareth, A., Newcombe, N. S., Shipley, T. F., Velazquez, M., & Weisberg, S. M. (2019). Beyond small-scale spatial skills: Navigation skills and geoscience education. *Cognitive Research: Principles and Implications*, 4, 17. <http://dx.doi.org/10.1186/s41235-019-0167-2>
- Newcombe, N. S., & Shipley, T. F. (2015). Thinking about spatial thinking: New typology, new assessments. In J. S. Gero (Ed.), *Studying visual and spatial reasoning for design creativity* (pp. 179–192). Dordrecht, Netherlands: Springer. http://dx.doi.org/10.1007/978-94-017-9297-4_10
- Playful Learning Landscapes Action Network. (2019). *Playful learning landscapes*. Retrieved from <https://playfullearninglandscapes.com/>
- Polinsky, N., Perez, J., Grehl, M., & McCrink, K. (2017). Encouraging spatial talk: Using children's museums to bolster spatial reasoning. *Mind, Brain, and Education*, 11, 144–152. <http://dx.doi.org/10.1111/mbe.12145>
- Pruden, S. M., & Levine, S. C. (2017). Parents' spatial language mediates a sex difference in preschoolers' spatial-language use. *Psychological Science*, 28, 1583–1596. <http://dx.doi.org/10.1177/0956797617711968>
- Pruden, S. M., Levine, S. C., & Huttenlocher, J. (2011). Children's spatial thinking: Does talk about the spatial world matter? *Developmental Science*, 14, 1417–1430. <http://dx.doi.org/10.1111/j.1467-7687.2011.01088.x>
- Quinn, P. C., & Liben, L. S. (2008). A sex difference in mental rotation in young infants. *Psychological Science*, 19, 1067–1070. <http://dx.doi.org/10.1111/j.1467-9280.2008.02201.x>
- Resnick, L., Verdine, B. N., Golinkoff, R., & Hirsh-Pasek, K. (2016). Geometric toys in the attic? A corpus analysis of early exposure to geometric shapes. *Early Childhood Research Quarterly*, 36, 358–365. <http://dx.doi.org/10.1016/j.ecresq.2016.01.007>

- Rosengren, K. S., Schein, S. S., & Gutiérrez, I. T. (2010). Individual differences in children's production of scale errors. *Infant Behavior and Development, 33*, 309–313. <http://dx.doi.org/10.1016/j.infbeh.2010.03.011>
- Sauter, M., Uttal, D. H., Alman, A. S., Goldin-Meadow, S., & Levine, S. C. (2012). Learning what children know about space from looking at their hands: The added value of gesture in spatial communication. *Journal of Experimental Child Psychology, 111*, 587–606. <http://dx.doi.org/10.1016/j.jecp.2011.11.009>
- Schacter, J., & Jo, B. (2017). Improving preschoolers' mathematics achievement with tablets: A randomized controlled trial. *Mathematics Education Research Journal, 29*, 313–327. <http://dx.doi.org/10.1007/s13394-017-0203-9>
- Stieff, M., Lira, M. E., & Scopelitis, S. A. (2016). Gesture supports spatial thinking in STEM. *Cognition and Instruction, 34*, 80–99. <http://dx.doi.org/10.1080/07370008.2016.1145122>
- Verdine, B. N., Bungler, A., Athanasopoulou, A., Golinkoff, R. M., & Hirsh-Pasek, K. (2017). Shape up: An eye-tracking study of preschoolers' shape name processing and spatial development. *Developmental Psychology, 53*, 1869–1880. <http://dx.doi.org/10.1037/dev0000384>
- Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2017). V. Results—Individual difference factors in spatial and mathematical skills. *Monographs of the Society for Research in Child Development, 82*, 81–88. <http://dx.doi.org/10.1111/mono.12284>
- Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., Newcombe, N. S., Flipowicz, A. T., & Chang, A. (2014). Deconstructing building blocks: Preschoolers' spatial assembly performance relates to early mathematical skills. *Child Development, 85*, 1062–1076. <http://dx.doi.org/10.1111/cdev.12165>
- Verdine, B. N., Lucca, K. R., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2016). The shape of things: The origin of young children's knowledge of the names and properties of geometric forms. *Journal of Cognition and Development, 17*, 142–161. <http://dx.doi.org/10.1080/15248372.2015.1016610>
- Verdine, B. N., Zimmermann, L., Foster, L., Marzouk, M. A., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. (2018). Effects of geometric toy design on parent-child interactions and spatial language. *Early Childhood Research Quarterly, 46*, 126–141. <http://dx.doi.org/10.1016/j.ecresq.2018.03.015>
- Voyer, D., Voyer, S. D., & Saint-Aubin, J. (2017). Sex differences in visual-spatial working memory: A meta-analysis. *Psychonomic Bulletin & Review, 24*, 307–334. <http://dx.doi.org/10.3758/s13423-016-1085-7>
- Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology, 101*, 817–835. <http://dx.doi.org/10.1037/a0016127>
- Weisberg, D. S., Hirsh-Pasek, K., & Golinkoff, R. M. (2013). Guided play: Where curricular goals meet a playful pedagogy. *Mind, Brain, and Education, 7*, 104–112. <http://dx.doi.org/10.1111/mbe.12015>
- Weisberg, D. S., Hirsh-Pasek, K., Golinkoff, R. G., Kittredge, A., & Klahr, D. (2016). Guided play: Principles and practices. *Current Directions in Psychological Science, 25*, 177–182. <http://dx.doi.org/10.1177/0963721416645512>
- Young, C., Cartmill, E., Levine, S., & Goldin-Meadow, S. (2014). Gesture and speech input are interlocking pieces: The development of children's jigsaw puzzle assembly ability. *Proceedings of the Annual Meeting of the Cognitive Science Society, 36*, 1820–1825. Retrieved from <https://escholarship.org/uc/item/3h2198h1>
- Zosh, J. M., Hirsh-Pasek, K., Hopkins, E. J., Jensen, H., Liu, C., Neale, D., . . . Whitebread, D. (2018). Accessing the inaccessible: Redefining play as a spectrum. *Frontiers in Psychology, 9*, 1124. <http://dx.doi.org/10.3389/fpsyg.2018.01124>
- Zosh, J. M., Verdine, B. N., Filipowicz, A., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2015). Talking shape: Parental language with electronic versus traditional shape sorters. *Mind, Brain, and Education, 9*, 136–144. <http://dx.doi.org/10.1111/mbe.12082>

Received September 3, 2019

Revision received April 30, 2020

Accepted May 1, 2020 ■